

Evaluation of predictions of strand stress at flexural strength for beams with unbonded or combined strands

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- This paper evaluates design code and literature-based methods for predicting strand stress at the nominal flexural capacity of prestressed concrete beams containing either combined (bonded and unbonded) strands or only unbonded strands.
- The authors compared prediction methods using experimental data from 39 beams with combined strands and 296 beams with only unbonded strands.
- The evaluation indicates that for beams with only unbonded strands, the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications* and other established prediction methods provide similar and accurate results, while assuming that strand stress equals the effective prestress is overly conservative and less accurate.
- Based on analysis of the available data, the equation for beams with bonded strands and the equation for beams with unbonded strands given in the *AASHTO LRFD specifications* 10th edition are recommended to achieve the most accurate predictions for beams with combined strands.

Unbonded strands are used in prestressed concrete bridge design when beam cross sections are thin and the number of strands is large enough to necessitate external prestressing or when strand replacement, monitoring,¹ or retensioning over the bridge's service life is desired. Unbonded strands are typically used with bonded mild reinforcement to prevent crack localization; however, there are cases where they can be used with bonded strands, creating a beam with combined (bonded and unbonded) strands. Examples include pretensioned beams with external prestressing or spliced pretensioned girders that have unbonded post-tensioning. Because there is no strain compatibility between concrete and unbonded strands, predicting the flexural strength of beams with only unbonded strands or beams with combined (bonded and unbonded) strands requires displacement compatibility; however, accurately predicting displacements of concrete beams nearing their capacity is challenging and requires a global analysis as opposed to the cross-sectional analyses performed for bonded strands. In addition, there are very limited test data on the flexural strength of beams with combined strands. The objective of this paper is to compile test data on beams with unbonded and combined steel strands and to use the compiled test data to evaluate the relative accuracies of flexural strength prediction methods for beams with unbonded tendons, with particular attention paid to beams with combined strands. Although studies have been conducted on beams with only unbonded strands or unbonded strands and nonprestressed reinforcement, there has been no such study evaluating beams with combined strands using test data from multiple studies. Beams with only bonded tendons

are outside the scope of this paper; there is already a large body of knowledge on these beams.^{2–6} Beams with nonmetallic strands are also out of scope because they are not as common as steel strands.

Flexural strength of prestressed concrete beams strongly depends on the strand stress at the beam nominal flexural capacity f_{ps} . As an alternative to performing global displacement compatibility analysis, design specifications (for example, the American Concrete Institute's *Building Code for Structural Concrete—Code Requirements and Commentary*⁷ [ACI CODE 318-25] and the 10th edition of the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design specifications*⁸) provide simple equations to predict f_{ps} in unbonded strands. These equations have been observed to predict f_{ps} conservatively when compared with test data for beams with only unbonded strands^{9–31} for varying support types,^{31–37} strand locations,^{38–42} and strand material.⁴³ Several studies have further evaluated these predictive equations. Harajli⁴⁴ used data from 176 tests, Au and Du⁴⁵ used data from 148 tests, Alqam et al.⁴³ used data from 227 tests, and Maguire et al.³⁶ used data from 253 tests and proposed modifications to better match test data and to capture the effects of continuity, strand location (external or internal), and nonmetallic strand material.

The majority of flexure tests used for evaluating predictive equations of f_{ps} were conducted on beams with only unbonded strands. Some test specimens contained nonprestressed mild reinforcement to ensure ductility and spread of plasticity; however, because bonded prestressing steel has smaller elongation capacity compared with mild reinforcement,^{46,47} the behavior of beams with combined strands may differ from that of beams with unbonded strands and mild reinforcement.

A few equations were proposed in the literature to predict f_{ps} in combined strands that include either carbon-fiber-reinforced-polymer strands,^{48,49} steel strands,^{46,47,50,51} or both,^{40,52–54} with varying levels of complexity. The AASHTO LRFD specifications⁸ conservatively assumes that f_{ps} in unbonded strands is equal to the effective prestress f_{pe} for beams with combined strands, leading to underestimation of beam flexural strength. Brenkus⁴⁷ and Consolazio et al.⁵¹ have evaluated the AASHTO LRFD specifications method and proposed predictive equations for combined strands based on their own test data. To evaluate the accuracy of various predictive methods, it is important to compare the predictions with test data from multiple research programs. This paper presents test databases, a detailed literature review, and predictive equations for beams with only unbonded strands and for beams with combined strands under flexure. The paper evaluates methods to predict f_{ps} by comparing predictions from various methods to test data from large datasets.

Research significance

This study offers a comprehensive review of strand stress prediction methods for beams with unbonded strands. The

focus is on beams with combined strands because this is an understudied topic leading to overconservatism and oversimplification of design codes. A test database for beams with unbonded strands has been established, expanding upon existing collections. The newly established database includes 43 more tests than the existing one created by Maguire et al.³⁶ for beams with only unbonded strands. In addition, a new test database for beams with combined strands has been compiled. To the authors' best knowledge, none of the existing test databases have focused on combined strands, and the existing predictive methods for combined strands have not been evaluated using test data from various research programs. This study provides a comparison of selected predictive methods using the newly compiled datasets to evaluate the accuracy of and variability in the predictions. The methods evaluated are selected so that they represent different approaches and are compatible with AASHTO LRFD specifications for potential implementation.

Predicting stress in unbonded strands

Stress in unbonded strands at the nominal flexural strength of a prestressed concrete beam is either assumed to be the same as the effective prestress or determined through methods that are based on a bond reduction coefficient, a collapse mechanism, or empirical equations. These methods were developed for beams with only unbonded strands that are typically reinforced with mild steel.

Bond reduction coefficient model

Bond reduction coefficients allow displacement compatibility analyses to be reduced to strain compatibility analyses and unify predictions of stress for bonded (a coefficient of 1.0) and unbonded strands.^{40,55,56} Baker⁵⁷ proposed a bond reduction coefficient defined as the ratio of steel strain to surrounding concrete strain at the section with maximum moment. A value of 0.1 was recommended as a safe limit.⁵⁸ Since then, several researchers have proposed values for the coefficient depending on loading conditions and strand profiles^{59–61} or related the coefficient to the neutral axis depth.^{61–63} Instead of a constant coefficient, Naaman⁶⁴ derived theoretical bond reduction coefficients based on strain increments beyond a reference state corresponding to the dead load and effective prestress f_{pe} . The first edition of the AASHTO LRFD specifications⁶⁵ (published in 1994) used the bond coefficient method proposed by Naaman and Alkhairi⁵⁵ to predict f_{ps} in unbonded strands (Eq. [1]):

$$f_{ps} = f_{pe} + \Omega_u E_{ps} \epsilon_{cu} \left(\frac{d_{ps}}{c} - 1 \right) \frac{L_1}{L_2} \leq 0.94 f_{py} \quad (1)$$

where

Ω_u = bond reduction coefficient

E_{ps} = modulus of elasticity of prestressing steel

ϵ_{cu} = ultimate compressive strain of concrete

d_{ps} = distance from the extreme compression fiber to the centroid of all strands

c = neutral axis depth

L_1 = length of loaded span affected by strand

L_2 = length of strand between end anchorages

f_{py} = yield strength of prestressing steel

Equation (1) was derived from strain compatibility at maximum moment sections at ultimate strength. The bond reduction coefficient Ω_u was associated with the span-to-depth ratio L/d_{ps} and loading configuration (one-point or two-point loading). Results from 143 simply supported beam tests were used to determine the coefficient. The equation accounts for beam continuity over several supports by introducing a factor based on span lengths L_1/L_2 . Other equations for Ω_u proposed in the literature^{40,41,43,66–69} are summarized in **Table 1**.

Table 1. Bond reduction coefficients for unbonded strands

Method	Loading configuration	Bond reduction coefficient Ω_u
Naaman and Alkhairi (1991) and Naaman et al. (2002)	1 point	$1.5/(L/d_{ps})$
Naaman and Alkhairi (1991) and Naaman et al. (2002)	2 point or uniform	$3.0/(L/d_{ps})$
Mutsuyoshi et al. (1995)	1 point	$4.36/(L/d_{ps}) - 0.084(S_d/L)$
Mutsuyoshi et al. (1995)	2 point or uniform	$1.47 + 10.3(L_d/L)/(L/d_{ps}) - 0.29(L_d/L)(S_d/L)$
Ng (2003)	All	$[0.895 - 1.364(Z/L)](d_{ps0}/h) - k_s$ where $k_s = 0.0096(S_d/d_{ps0})$ when $S_d/d_{ps0} \leq 15$ $k_s = 0.144$ when $S_d/d_{ps0} > 15$
Sivaleepunth et al. (2006)	All	$0.3(L_d/L) + 0.01(S_d/d_{ps}) + 0.1$
He and Liu (2010)	1 point	$\frac{0.00126 \left(0.3 + \frac{3}{\frac{L}{d_{ps}}} \right) e_m R_s}{(d_{ps} - c) \epsilon_{cu}}$ where $R_s = 1 - \frac{\phi \Delta}{\eta}$
He and Liu (2010)	2 point or uniform	$\frac{0.00162 \left(0.78 + \frac{2.35}{\frac{L}{d_{ps}}} \right) e_m R_s}{(d_{ps} - c) \epsilon_{cu}}$ where $R_s = 1 - \frac{\phi \Delta}{\eta}$
Alqam et al. (2021)	1 point	$e_m/d_{ps} [0.02 + 3.6/(L/d_{ps})]$
Alqam et al. (2021)	2 point or uniform	$e_m/d_{ps} [0.29 + 6.48/(L/d_{ps})]$

Note: c = neutral axis depth; d_{ps} = distance from the extreme compression fiber to the centroid of all strands; d_{ps0} = initial distance from the extreme compression fiber to the centroid of external strands; e_m = strand eccentricity at midspan; h = beam depth; k_s = constant to account for second-order effects by external strands; L = span length; L_d = distance from beam support to deviation point; R_s = stress increase reduction factor to account for second-order effects by external strands; S_d = distance between two symmetrically placed deviators; Z = shear span; Δ = span deflection; ϵ_{cu} = ultimate compressive strain of concrete; η = parameter based on loading type and strand profile; ϕ = parameter based on strand type in He and Liu (2010) study; Ω_u = bond reduction coefficient.

Collapse mechanism model

MacGregor et al.⁷⁰ introduced a collapse mechanism model that idealizes beams with unbonded strands as a series of rigid elements connected with plastic hinges formed at the extreme compression fiber after cracking. In this model, hinge locations need to be identified when beams are continuous. The model in **Fig. 1** illustrates this concept for a simply supported beam with unbonded draped strands. The changes in length of the strand at a plastic hinge at midspan can be calculated using geometry if the strand is assumed to elongate only at the hinge. Using geometry, strain increase in unbonded strands can be expressed according to Eq. (2).

$$\Delta\epsilon_{ps} = \frac{\delta}{L} = \frac{(d_{ps} - c)}{L} \quad (2)$$

where

- $\Delta\epsilon_{ps}$ = strain increase in prestressing strands
- δ = change in strand length at a hinge location
- L = span length
- θ = angle of rotation at the hinge

The angle of rotation at the hinge θ in Eq. (2) is limited by the rotational capacity of the beam. Various methods have been proposed to estimate this angle. The angle has been associated with deflection Δ ^{39,71} (Fig. 1), the curvature ϕ in Eq. (3) along with the plastic hinge length L_p ^{33,44} or a scaled plastic hinge length ψ ^{14,16,34,36,70} as Eq. (3) shows.

$$\theta = \frac{4\Delta}{L} = \int \phi(x) dx = L_p \phi = L_p \frac{\epsilon_{cu}}{c} = \psi \epsilon_{cu} \quad (3)$$

where

- L_p = plastic hinge length

The scaled plastic hinge length ψ is typically an empirical constant. Maguire et al.³⁶ proposed modifications to ψ considering continuity (simply supported or continuous beams) and strand location (internal or external). They recommended that ψ be derived analytically instead of using constant values.

The 10th edition of the AASHTO LRFD specifications⁸ follows the collapse mechanism model. The equation assumes a constant ψ of 10.5 based on Tam and Pannell,¹⁴ E_{ps} of 197 GPa (28,500 ksi), and ϵ_{cu} of 0.003. The AASHTO LRFD specifications⁸ equation that combines Eq. (2) and (3) and Hooke's law is given in Eq. (4).

$$f_{ps} = f_{pe} + 6200 \left(\frac{d_{ps} - c}{L_e} \right) \leq f_{py} \text{ (MPa)} \quad (4)$$

$$f_{ps} = f_{pe} + 900 \left(\frac{d_{ps} - c}{L_e} \right) \leq f_{py} \text{ (ksi)}$$

where

- L_e = effective strand length = $\frac{2L_i}{2 + N_s}$
- L_i = strand length between anchorages
- N_s = number of plastic hinges at support in an assumed failure mechanism crossed by the strand between anchorages

Au and Du⁴⁵ proposed the expression shown in Eq. (5) to calculate f_{ps} in unbonded strands. Based on comparisons of predictions with test results of 148 simply supported beams with internal or external strands, ψ was proposed to be 9.3. Au and Du⁴⁵ also recommended omitting the $\psi E_{ps} A_{ps} \epsilon_{cu} / 0.85 \beta_1 f'_c b_w L$ term that ranges from 0.01 to 0.05 in most cases.

$$f_{ps} = f_{pe} + \frac{\psi E_{ps} A_{ps} \epsilon_{cu} (d_{ps} - c_e)}{\left(1 + \frac{\psi E_{ps} A_{ps} \epsilon_{cu}}{0.85 \beta_1 f'_c b_w L} \right)} \leq f_{py} \quad (5)$$

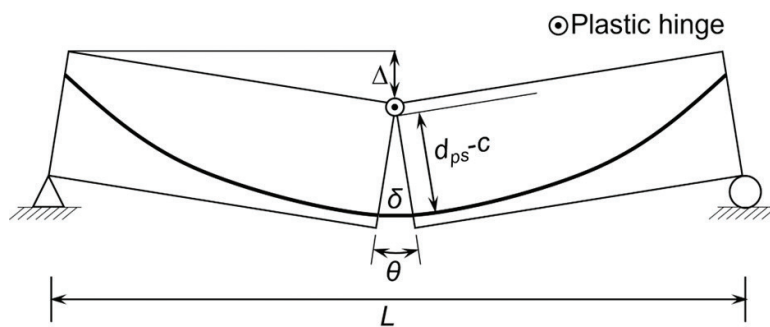


Figure 1. Collapse mechanism geometry.

Source: Adapted from MacGregor et al., 1989.

Note: c = neutral axis depth; d_{ps} = distance from the extreme compression fiber to the centroid of strands; L = span length; δ = change in strand length at a hinge location; Δ = span deflection; θ = angle of rotation at the hinge.

where

- c_e = neutral axis depth at ultimate assuming f_{ps} in unbonded strands is equal to f_{pe}
- A_{ps} = prestressing strand area
- β_1 = concrete compressive stress block reduction factor
- f'_c = compressive strength of concrete
- b_w = beam web width

Haraji³³ argued that the collapse mechanism for continuous beams depends on load patterns and suggested the use of the plastic hinge length L_p with the expression given by Eq. (6). An expression to compute f_{ps} in unbonded strands with L_p was also developed,⁴⁴ as Eq. (7) shows. The equation agreed well with 176 test data consisting of 161 simply supported and 15 continuous beams.

$$L_p = \left(\frac{20.7}{f} + 10.5 \right) c \quad (6)$$

where

- f = ratio of span length to distance between loading points; f is taken as ∞ for a single concentrated load, 3 for two point-loads applied at third points across a span, and 6 for uniform loading

$$f_{ps} = f_{pe} + \frac{420 N_p}{\frac{L_t}{d_{ps}}} \left(1 - \frac{c_e}{d_{ps}} \right) \leq 0.95 f_{py} \quad (\text{MPa})$$

$$f_{ps} = f_{pe} + \frac{N_p E_{ps} \epsilon_{cu}}{\frac{L_t}{d_{ps}}} \left(1 - \frac{c_e}{d_{ps}} \right) \leq 0.95 f_{py} \quad (\text{ksi}) \quad (7)$$

where

- N_p = load parameter = $\left(\frac{20.7}{f} + 10.5 \right) n_p^+ + 10.5 n_p^-$
- n_p^+ = the number of positive moment plastic hinges
- n_p^- = the number of negative moment plastic hinges

Empirical models

The equation in ACI 318-25⁷ (Eq. [8]) was statistically developed based on work by Mattock et al.¹³ with limited test data at the time and was modified by Mojtahedi and Gamble.⁷² The equation is valid for $f_{pe} \geq 0.5 f_{pu}$, where f_{pu} is the ultimate strength of the prestressing strands. Mojtahedi and Gamble⁷² observed that f_{ps} in unbonded strands of slender beams, especially with span to depth ratios L/d_{ps} above 45, developed at a slower rate than the one predicted by Mattock et al.¹³ They introduced a constraint of L/d_{ps} into

the prediction as Eq. (8) shows; however, this piecewise representation of L/d_{ps} does not capture the continuous change of f_{ps} relative to L/d_{ps} . In addition, Eq. (8) was derived without considering the presence of mild reinforcement⁴³ and does not consider span continuity.

$$f_{ps} = f_{pe} + 70 + \frac{f'_c}{\frac{100 A_{ps}}{b d_{ps}}} \leq f_{pe} + 420 \text{ or } f_{py} \text{ for } \frac{L}{d_{ps}} \leq 35 \quad (\text{MPa})$$

$$f_{ps} = f_{pe} + 10 + \frac{f'_c}{\frac{100 A_{ps}}{b d_{ps}}} \leq f_{pe} + 60 \text{ or } f_{py} \text{ for } \frac{L}{d_{ps}} \leq 35 \quad (\text{ksi})$$

$$f_{ps} = f_{pe} + 70 + \frac{f'_c}{\frac{300 A_{ps}}{b d_{ps}}} \leq f_{pe} + 200 \text{ or } f_{py} \text{ for } \frac{L}{d_{ps}} > 35 \quad (\text{MPa})$$

$$f_{ps} = f_{pe} + 10 + \frac{f'_c}{\frac{300 A_{ps}}{b d_{ps}}} \leq f_{pe} + 30 \text{ or } f_{py} \text{ for } \frac{L}{d_{ps}} > 35 \quad (\text{ksi}) \quad (8)$$

where

- b = beam width

Effective prestress assumption

The provisions of the *Canadian Highway Bridge Design Code*⁷³ (CSA S6:19) equate f_{ps} in unbonded strands to the effective prestress (lower-bound stress) as Eq. (9) shows unless a detailed analysis demonstrates a higher value can be used. This approach neglects any stress increase in unbonded strands under increasing external loading. While conservative, using the effective prestress may significantly underestimate flexural capacity.

$$f_{ps} = f_{pe} \quad (9)$$

Predicting stress in bonded strands

For beams with bonded strands, concrete and strands have strain compatibility. Therefore, stress increase in bonded strands due to external loading can be calculated through a cross-sectional analysis. As an alternative to strain compatibility, the AASHTO LRFD specifications⁸ and CSA S6:19⁷³ provide Eq. (10) for f_{ps} in bonded strands. The equation can be used when f_{pe} is more than half of the ultimate strength of the strands according to the AASHTO LRFD specifications and when c/d_{ps} is less than or equal to 0.5 according to CSA S6:19. The equation was developed by Loov⁴ and uses a simplified representation of strand stress-strain behavior to calculate stress from strains that develop at nominal moment capacity.

$$f_{ps} = f_{pu} \left(1 - k \frac{c}{d_{ps}} \right) \quad (10)$$

where

- k = factor to account for prestressing steel material

Predicting stress in combined strands

For beams with combined strands, strain and displacement compatibility approaches can be used together to predict strand stresses at the nominal flexure capacity. Strain compatibility is used for bonded strand stress f_{psb} , and displacement compatibility is used for unbonded strand stress f_{psu} . Design codes only offer simplified and generally conservative approaches for this strand configuration. In lieu of a detailed analysis, the AASHTO LRFD specifications⁸ allows a simplified approach in which f_{psb} is calculated using Eq. (10) and f_{psu} is assumed to be the effective prestress f_{pe} (the lower bound of stress in unbonded strands). The overall strand stress f_{ps} is calculated as the average of bonded and unbonded strands weighted according to their areas.

Brenkus⁴⁷ concluded that the simplified approach in the AASHTO LRFD specifications⁸ is insufficient for predicting flexural strength when using small areas of bonded strands. The research suggested a factor γ that reduces the unbonded strand stress calculated by Eq. (4). The modified AASHTO LRFD specifications equation is given in Eq. (11).

$$f_{psu} = \max \left[\gamma \left\{ f_{pe} + 6200 \left(\frac{d_{ps} - c}{L_e} \right) \right\}, f_{pe} \right] \text{ (MPa)}$$

$$f_{psu} = \max \left[\gamma \left\{ f_{pe} + 900 \left(\frac{d_{ps} - c}{L_e} \right) \right\}, f_{pe} \right] \text{ (ksi)} \quad (11)$$

where

γ = reduction factor to account for bonded and unbonded prestressing strand areas =

$$\frac{A_{psu} (d_{psu} - c)}{A_{psu} (d_{psu} - c) + A_{psb} (d_{psb} - c)}$$

A_{psu} = area of unbonded prestressing strands

d_{psu} = distance from the extreme compression fiber to the centroid of unbonded strands

A_{psb} = area of bonded prestressing strands

d_{psb} = distance from the extreme compression fiber to the centroid of bonded strands

Ozkul et al.⁷⁴ conducted an incremental analysis using idealized truss beam models. A closed-form solution was developed for f_{psu} assuming that strand stress would be within the linear-elastic range. Unal⁵³ extended the approach proposed by Ozkul et al.⁷⁴ to account for the effects of bonded strands when used with unbonded strands. Eq. (12) shows the equation proposed by Unal,⁵³ who reported that this equation correlated better with test data from 20 beams with bonded steel strands and unbonded steel or FRP strands compared with the AASHTO LRFD specifications⁸ and ACI 318-25.⁷ Abu-Obeidah⁵⁴ tested eight beams with combined strands and

verified that predictions by Eq. (12) had a correlation coefficient R of 0.99 with their experimental study.

$$f_{psu} = f_{pe} + \frac{E_{ps}}{1000} \left(\frac{\beta_1 f_c' b e_{ps}}{A_{psu} f_{psu} + A_{psb} f_{psb} + A_s f_y} \right) \leq f_{py} \quad (12)$$

where

e_{ps} = eccentricity of unbonded strands

A_s = area of mild steel reinforcement

f_y = yield strength of mild steel reinforcement

Nasreddine et al.⁷⁵ proposed an iterative analytical approach based on a simplified curvature idealization, equilibrium, and the conservation of energy that is applicable to steel and non-metallic strands and achieved acceptable accuracy compared with their test data. In a related study, Harajli et al.⁷⁶ simplified a strain compatibility approach with a series of assumptions, which included assigning an approximate stress for bonded strands at ultimate flexural capacity when calculating the neutral axis depth, to develop equations suitable for design. Brenkus⁴⁷ and Consolazio et al.⁵¹/Frosch et al.⁷⁷ suggested modifications to the simplified approach in the AASHTO LRFD specifications,⁸ calling attention to potential for strand fracture at lower quantities of bonded reinforcement. Consolazio et al.⁵¹/Frosch et al.⁷⁷ recommended calculating f_{psu} and f_{psb} according to Eq. (4) and Eq. (10), respectively. Consolazio et al.⁵¹ proposed a minimum area of bonded strands to prevent fracture of bonded strands and concluded that their proposed approach was more accurate than the simplified approach in the AASHTO LRFD specifications⁸ and remained conservative. Frosch et al.⁷⁷ recommended a minimum area of bonded reinforcement.

Experimental studies

To evaluate the prediction methods previously summarized, a test database was created of 296 beams with unbonded strands and 39 beams with combined strands. All beams in the database had prestressing steel that ranged from 1034 to 2068 MPa (150 to 300 ksi) in design ultimate strength and the beams were either simply or continuously supported. Statistical distributions of the features of the tested beams in the database are analyzed in this section. Throughout the paper, strand location refers to internal or external strands.

Database for beams with unbonded strands

A test database of 296 beams with unbonded strands that failed under flexure was compiled from 32 studies^{9,13–18,20–25,29,37,42,54,63,66,68,69,71,74,77–85} as summarized in **Table 2**.

Database for beams with combined strands

A database of 39 beams with combined strands was compiled from nine studies^{26,42,50,51,54,66,77,83,86} and is summarized

Table 2. Flexure tests on beams with only unbonded strands

Reference	Number of specimens	Continuity	Unbonded strand location	Beam depth, mm	Span length, m	L/d_{ps}	Number of loads along span
Janney et al. (1956)	7	Simply supported	Internal	305	2.7	13	2
Pannell (1969)	38	Simply supported	Internal	152 to 228	1.8 to 4.1	11 to 53	1
Mattock et al. (1971)	7	Simply supported and continuous	Internal	305	8.5	34	4
Tam and Pannell (1976)	8	Simply supported	Internal	158 to 230	3.0 to 4.1	18 to 43	1
Burns et al. (1978)	2	Continuous	Internal	70	3.0	53	Uniform
Cooke et al. (1981)	9	Simply supported	Internal	180	2.2 to 4.6	18 to 38	2
Tao and Du (1985)	20	Simply supported	Internal	280	4.2	19	2
Burns et al. (1991)	3	Continuous	Internal	356 to 559	7.6	16 to 27	1
Campbell and Chouinard (1991)	6	Simply supported	Internal	280	3.3	15	2
Harajli and Kanj (1991)	26	Simply supported	Internal	229 to 330	2.0 to 3.0	8 to 19	1 to 2
Gauvreau (1992)	7	Simply supported	Internal	770	10	15	1 to 9
Zhang et al. (1993)	7	Simply supported	External	450	5	10 to 14	1 to 2
Chakrabarti et al. (1994)	33	Simply supported	Internal	83 to 229	3.5	18 to 55	2
Mutsuyoshi et al. (1995)	4	Simply supported	Internal or external	325	6.1	24	2
Aravinthan et al. (1995)	1	Continuous	External	325	4.1	18	2
Yaginuma (1995)	9	Simply supported	External	240 to 400	2.7 to 4	15 to 17	2
Tan and Ng (1997)	6	Simply supported	External	300	3	12 to 15	2
Kosa et al. (1997)	2	Simply supported	External	1000	10	11	2
Nishikawa et al. (2000)	4	Simply supported	Internal or external	500	7.5	19	2
Tan et al. (2001)	4	Simply supported	External	300	4.5	23	2
Aparicio et al. (2002)	6	Simply supported and continuous	External	600	7.2	14	2
Harajli et al. (2002)	8	Continuous	External	200	1.5	9 to 15	1
Ng (2003)	12	Simply supported	External	300	1.5 to 6	6 to 30	2
Sivaleepunth (2006)	3	Simply supported	Internal	300	3	15	2
Ozkul et al. (2008)	21	Simply supported	Internal	229 to 254	1.6 to 3.0	10 to 18	2
Decheng (2009)	5	Continuous	External	280	2.8	12 to 17	1 to 2
Vu et al. (2010)	2	Simply supported	Internal	200	3	21	2
Yoo and Ha (2010)	19	Simply supported	External	480 to 600	3 to 7.5	10 to 20	2
Six (2015)	4	Simply supported	Internal	132	5.9	53	2
Maguire et al. (2016)	3	Continuous	Internal	190	6.1	38	Uniform
Abu-Obeidah (2017)	3	Simply supported	Internal	254	3.0	15 to 20	2
Frosch et al. (2025)	2	Simply supported	Internal or internal and external	711	7.9	10 to 14	2

Note: Prestressing steel was strands, wires, or bars. d_{ps} = distance from the extreme compression fiber to the centroid of all strands; L = span length. 1 mm = 0.0394 in.; 1 m = 3.281 ft.

Table 3. Flexure tests on beams with combined strands

Reference	Number of specimens	Continuity	Unbonded strand location	Beam depth, mm	Span length, m	L/d_{ps}	Number of loads along span
Mutsuyoshi et al. (1995)	1	Simply supported	External	325	6.1	24	2
Kosa et al. (1997)	4	Simply supported	External	1000	10	11	2
Aravinthan et al. (2005)	7	Simply supported and continuous	External	150	5	19 to 26	2
Yoo and Ha (2010)	2	Simply supported	External	380	3.0	8	2
Brenkus et al. (2017)	2	Simply supported	Internal	1575	11.9	8	1 to 2
Abu-Obeidah (2017)	4	Simply supported	Internal	254	3.0	15 to 18	2
Abu-Saibia (2018)	8	Continuous	Internal	254	3.0	15	2
Consolazio et al. (2022)	7	Simply supported and overhang	Internal	1118	13.7 to 22.9	15 to 24	1
Frosch et al. (2025)	4	Simply supported	Internal or external	711	7.9	13 to 14	2

Note: d_{ps} = distance from the extreme compression fiber to the centroid of all strands; L = span length. 1 mm = 0.0394 in.; 1 m = 3.281 ft.

in **Table 3**. Three of the seven beams tested by Consolazio et al.⁵¹ had overhangs to produce negative moment. In our study, the capacities of these three beams were calculated assuming a single hinge was formed at failure (concrete crushing or strand fracture) similar to simply supported beams. Therefore, these beams are presented with the simply supported beams. Overall, test data for beams with combined strands were very limited (39 tests) compared with beams with only unbonded strands (296 tests).

Features of the beams within databases

Figure 2 presents the number of beams according to cross-section shape, span-to-depth ratio L/d_{ps} , concrete compressive strength f'_c , ratio of unbonded strand area to the effective concrete area A_{psu}/bd_{psu} , and ratio of unbonded strand area to total reinforcement area. Plots on the left are for beams with only unbonded strands, while plots on the right are for beams with combined strands. For beams with only unbonded strands, the total reinforcement area is the sum of the unbonded strand and mild reinforcement areas ($A_{psu} + A_s$). For beams with combined strands, the total reinforcement area is the total strand area ($A_{psb} + A_{psu}$). Concrete compressive strength was taken as the measured strength when reported and as the design strength when the measured strength was not reported.

The cross-section data in **Fig. 2** shows that most beams with unbonded strands have rectangular and T-shaped cross-sections, while most beams with combined strands have I-shaped cross-sections that are common for bridge superstructures. The ratio L/d_{ps} comparison in **Fig. 2** indicates that most tests were conducted on beams with L/d_{ps} between 10 and 30, covering common span-to-depth ratios for building and bridge

superstructures. The largest L/d_{ps} ratio was 55 for beams with unbonded strands and was 26 for beams with combined strands. The concrete compressive strength data in **Fig. 2** indicates that most beams with only unbonded strands had concrete compressive strengths from 30 to 50 MPa (4 to 7 ksi), which is common for building beams but lower than is used for bridge girders. Beams with combined strands had concrete compressive strengths greater than 50 MPa (7 ksi). The ratio of unbonded strand area to the effective concrete area data of **Fig. 2** indicates that most beams had values of A_{psu}/bd_{psu} less than 1.0%. Additional data on beams with A_{psu}/bd_{psu} greater than 1.0% may be needed because increasing A_{psu}/bd_{psu} was reported to decrease f_{ps} .¹⁶

For beams with only unbonded strands, the ratio of unbonded strand area to total reinforcement area data of **Fig. 2** shows that most beams contained mild reinforcement. Out of 296 beams with unbonded strands, 75 beams had no mild reinforcement

$$\left(\frac{A_{psu}}{A_{psu} + A_s} = 100\% \right).$$

These beams may exhibit tied arch behavior⁷² or experience brittle failures. For beams with combined strands, **Fig. 2** shows that the values of unbonded strand area to total strand area ratios were between 20% and 90%.

The majority of test specimens (97% for beams with unbonded strands and 66% for beams with combined strands) had span lengths less than 9 m (30 ft) as presented in **Tables 2** and **3**. Only 24% of beams with unbonded strands and only 5% of beams with combined strands were tested under one point loading along the span. Most beams were tested under two-point loading across the span. The number of loading points and span length are important because they affect unbonded strand stress at ultimate flexural capacity.

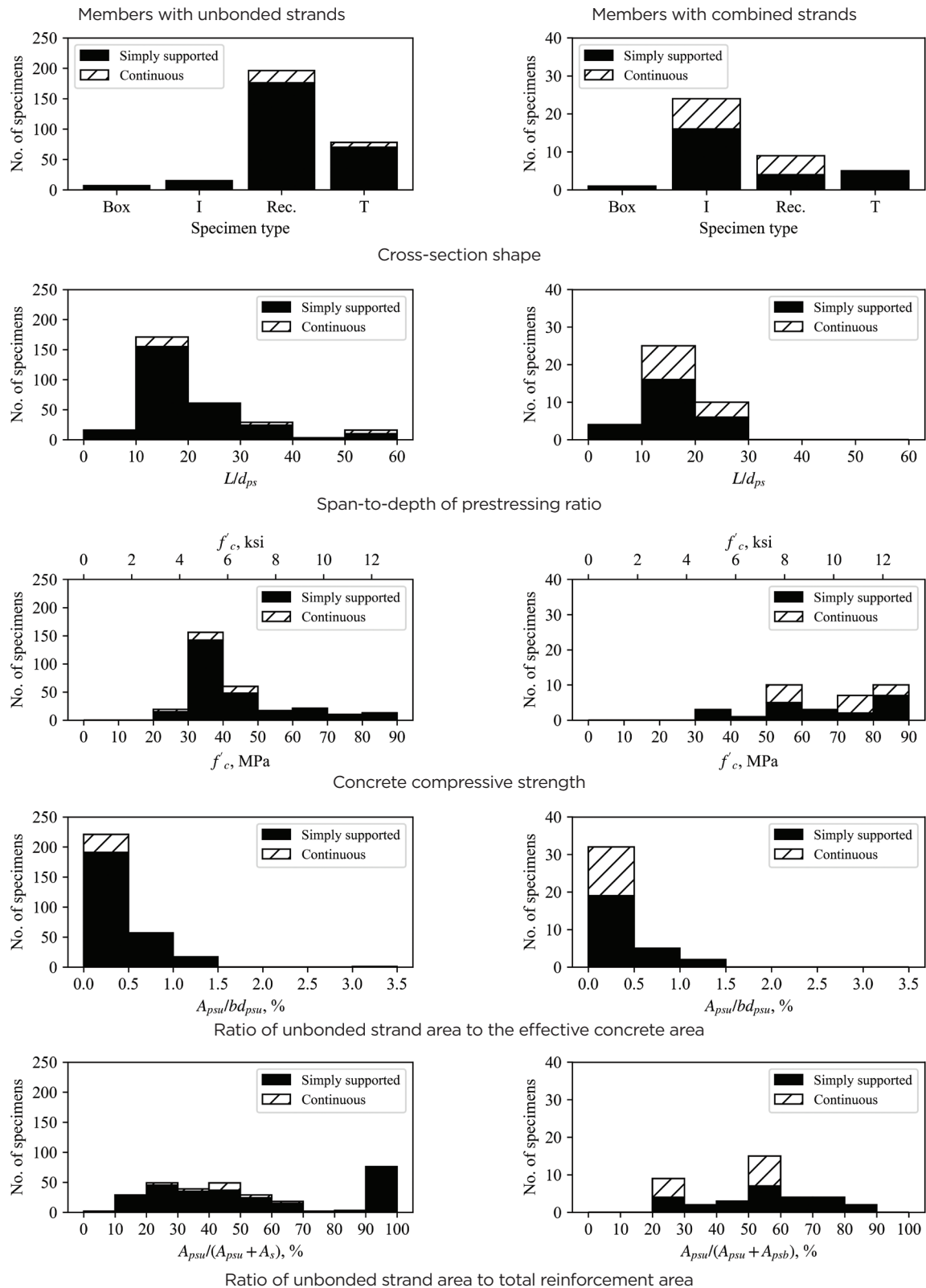


Figure 2. Histograms of test data for beams with unbonded or combined strands.

Note: A_{psb} = area of bonded prestressing strands; A_{psu} = area of unbonded prestressing strands; A_s = area of mild steel reinforcement; b = beam width; d_{ps} = distance from the extreme compression fiber to the centroid of all strands; d_{psu} = distance from the extreme compression fiber to the centroid of unbonded strands; f'_c = compressive strength of concrete; L = span length; No. = number; Rec. = rectangular. 1 MPa = 0.145 ksi.

Evaluation of prediction methods

Prediction methods were evaluated by comparing the predicted f_{ps} with the experimentally measured f_{ps} . The parameter f_{ps} was selected for these comparisons to determine which expressions can best estimate the strand stress at nominal capacity. The parameter f_{ps} was also chosen because it is directly used in design calculations to calculate nominal beam flexural strength.

For predicting f_{ps} in beams with only unbonded strands, Eq. (4), (1), (8), and (9) provided by four design specifications (2024⁸ and 1994⁶⁵ AASHTO LRFD specifications, ACI 318-25,⁷ and CSA S6:19⁷³) were evaluated using the test database. The 2024⁸ and 1994⁶⁵ AASHTO LRFD specifications were selected to evaluate the collapse mechanism and bond reduction models, respectively. The ACI 318-25⁷ and CSA S6:19⁷³ methods were selected to evaluate the empirical model and the assumption that f_{ps} equals f_{pe} , respectively.

For predicting f_{ps} of unbonded strands in beams with combined strands, approaches provided by the 2024 AASHTO LRFD specifications,⁸ Brenkus,⁴⁷ Unal,⁵³ and Consolazio et al.⁵¹/Frosch et al.⁷⁷ were evaluated.

Statistical measures used

The following statistical measures were used to evaluate the equations:

- the ratio of the experimentally measured f_{ps} to the predicted f_{ps} or ratio = $\frac{f_{ps}(\text{measured})}{f_{ps}(\text{predicted})}$ (the ratio from hereon)
- the mean of the ratio (the mean from hereon)
- the coefficient of correlation R between the predicted and experimentally measured f_{ps}
- the root mean square error $RMSE$ of the predictions
- the percentage of the ratios less than 1.0 (%Ratio < 1.0)

In this study, the predictions are referred to as conservative if the ratio is larger than 1.0, indicating the measured capacity is larger than the predicted capacity. The mean provides a measure of error in an average sense as it averages the ratios for conservative (ratios > 1.0) and unconservative (ratios < 1.0) predictions. The lower the $RMSE$, the closer the predictions are to the experimentally measured values. When predicted and experimentally measured f_{ps} are plotted together, a zero $RMSE$ would mean all values are on a 45-degree line. The R value measures the correlation strength of predicted compared with test data relative to a best-fit line, but not necessarily relative to the 45-degree line. A high R value with low $RMSE$ indicates high accuracy coupled with high correlation. The %Ratio < 1.0 is the fraction of unconservative predictions; higher %Ratio < 1.0 means a larger number of unconservative predictions.

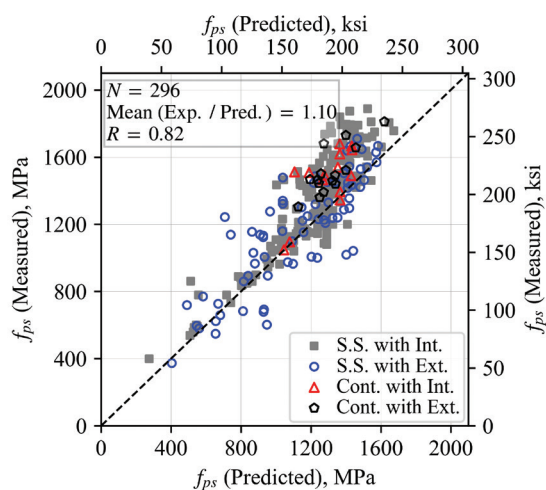
Predicting stress in beams with unbonded strands

Figure 3 presents f_{ps} measured by experiments and predicted by the 2024⁸ and 1994⁶⁵ AASHTO LRFD specifications, ACI 318-25,⁷ and CSA S6:19⁷³ for beams with only unbonded strands. Data points above the 45-degree line indicate that measurements were greater than predictions and hence the predictions were conservative. In this figure, a distinction is made between simply supported (S.S.) and continuous beams and between internal and external strands. ACI 318-25⁷ allows the use of Eq. (8) if the effective prestress f_{pe} is greater than half of the ultimate strength of strand f_{pu} . Therefore, test data of beams with $f_{pe} < 0.5f_{pu}$ were excluded when evaluating the ACI 318-25 method. The number of test specimens, mean, and R values are provided in Fig. 3 for all test data regardless of support type and strand location.

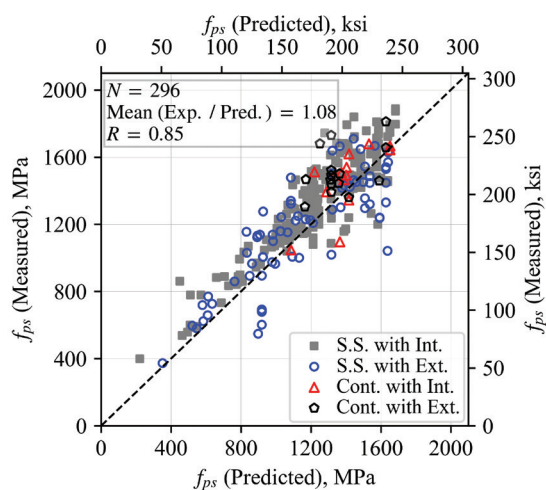
The values of the mean were similar for the two AASHTO LRFD specifications editions and ACI 318-25. CSA S6:19 had the highest mean, indicating, as expected, that when f_{ps} is assumed to be equal to f_{pe} , the predictions are significantly more conservative compared to the other methods. It is interesting that the values of R were similar for all prediction methods regardless of the level of complexity of the methods. This indicates that f_{ps} may be governed by f_{pe} (the lower bound of f_{ps} in unbonded strands) for most specimens due to the relatively large f_{pe} compared with the stress increase beyond f_{pe} .

Table 4 presents the statistical measures for prediction accuracy for each method for each beam support type (simply supported or continuous) and strand location (internal or external). Statistical measures are provided for all continuous beams regardless of the strands being internal or external due to the small number of continuous specimens in the database. An asterisk indicates the type of beam for which the prediction methods have the least favorable statistical measure.

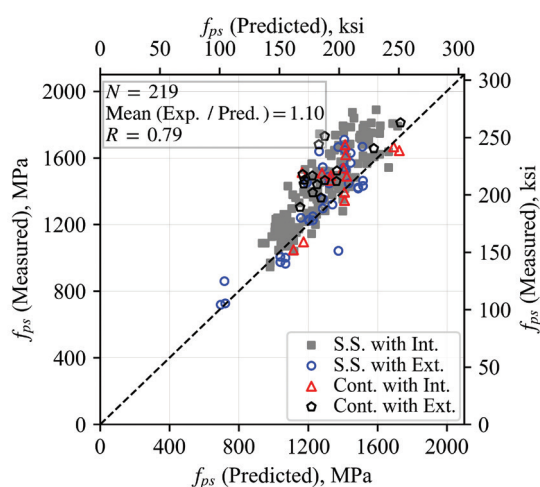
When specimens with specific support conditions or strand locations were considered, the mean values for all prediction methods were greater than 1.0, indicating that the methods can produce averagely conservative predictions for any support condition and strand location. While external strands are theoretically susceptible to eccentricity loss due to second-order effects,⁵⁶ the statistical evaluation showed that f_{ps} is not very sensitive to strand location. This empirical uniformity may stem from practical detailing of test specimens such as placing deviators at critical sections and effectively mitigating local second-order eccentricity losses. For continuous beams, the 2024⁸ and 1994⁶⁵ AASHTO LRFD specifications and ACI 318-25⁷ had relatively low R values and relatively high $RMSE$ values, indicating poorer predictions compared with ones for simply supported beams. The %Ratio < 1.0 values of these methods were larger (>32%) for simply supported beams with external strands, indicat-



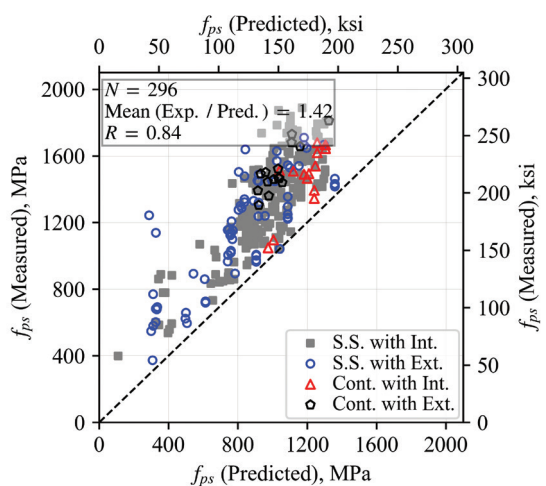
AASHTO LRFD specifications (2024)



AASHTO LRFD specifications (1994)



ACI 318-25



CSA S6:19

Figure 3. Predicted compared with measured strand stress f_{ps} for beams with unbonded strands.

Note: AASHTO LRFD specifications (2024) = the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*, 10th edition; AASHTO LRFD specifications (1994) = *AASHTO LRFD Bridge Design Specifications*, 1st edition; ACI 318-25 = the American Concrete Institute's *Building Code for Structural Concrete—Code Requirements and Commentary* (ACI CODE-318-25); Cont. = continuous; CSA S6:19 = the Canadian Standards Association (CSA) Group's *Canadian Highway Bridge Design Code*; Exp. = experimental; Ext. = external strands; Int. = internal strands; N = number of data points; Pred. = predicted; R = correlation coefficient; S.S. = simply supported. 1 MPa = 0.145 ksi.

ing less conservatism compared with continuous beams and compared with simply supported beams with internal strands. Because CSA S6:19⁷³ assumes f_{ps} equals f_{pe} , its predictions of all specimens with any support or strand location were conservative (%Ratio < 1.0 = 0%).

Predicting stress in beams with combined strands

Stress in unbonded strands for beams with combined strands f_{psu} For beams with combined strands, predictions of unbonded strand stress by the 2024 AASHTO LRFD specifications,⁸ Brenkus,⁴⁷ Unal,⁵³ and Consolazio et al.⁵¹/Frosch

et al.⁷⁷ were evaluated. **Figure 4** shows f_{psu} as measured by experiments and as predicted. Datapoints above the 45-degree line indicate conservative predictions. The 2024 AASHTO LRFD specifications⁸ and Brenkus⁴⁷ methods had high mean values (1.94 and 1.56) and %Ratio < 1.0 (0%), demonstrating that the methods are very conservative. The method proposed by Unal,⁵³ on the other hand, had a mean of 0.96 and the lowest $RMSE$ (222 MPa [32 ksi]), showing that the method is more accurate, but it is less conservative than the 2024 AASHTO LRFD specifications⁸ and Brenkus⁴⁷ methods with 67% of the predictions being greater than the measured values. The prediction method by Consolazio et al.⁵¹/Frosch et al.⁷⁷ with a mean of 1.21 and an R of 0.49 shows that the

Table 4. Statistical measures for the predictions of stress in strands at nominal flexural capacity f_{ps} of unbonded strands for beams with only unbonded strands

	Design specifications	Simply supported with internal strands	Simply supported with external strands	Continuous with internal or external strands	All specimens
Number of tests	n/a	193	73	30	296
Mean (experimental/predicted)	2024 AASHTO LRFD BDS	1.10	1.06	1.19*	1.10
	1994 AASHTO LRFD BDS	1.10	1.03	1.12*	1.08
	ACI 318-25	1.10	1.05	1.15*	1.10
	CSA S6:19	1.40	1.50*	1.36	1.42
Correlation coefficient R	2024 AASHTO LRFD BDS	0.88	0.84	0.15*	0.82
	1994 AASHTO LRFD BDS	0.91	0.83	0.30*	0.85
	ACI 318-25	0.86	0.85	0.45*	0.79
	CSA S6:19	0.83	0.80*	0.84	0.84
Root mean square error $RMSE$, MPa	2024 AASHTO LRFD BDS	184	185	391*	215
	1994 AASHTO LRFD BDS	163	192	332*	194
	ACI 318-25	176	157	333*	202
	CSA S6:19	399	396	439*	401
%Ratio < 1.0, %	2024 AASHTO LRFD BDS	18	38*	3	21
	1994 AASHTO LRFD BDS	16	42*	20	23
	ACI 318-25	10	32*	20	14
	CSA S6:19	0	0	0	0

Note: %Ratio < 1.0 = percentage of the ratios of the experimentally measured f_{ps} to the predicted f_{ps} that are less than 1.0; 1994 AASHTO LRFD BDS (1994) = the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*, first edition; 2024 AASHTO LRFD BDS = *AASHTO LRFD Bridge Design Specifications*, 10th edition; ACI 318-25 (2025) = the American Concrete Institute's *Building Code for Structural Concrete—Code Requirements and Commentary* (ACI CODE-318-25); CSA S6:19 (2019) = the Canadian Standards Association (CSA) Group's *Canadian Highway Bridge Design Code*; n/a = not applicable. 1 MPa = 0.145 ksi.

* In each row, the type of beam for which the prediction method has the least favorable statistical measure.

use of the 2024 AASHTO LRFD specifications⁸ provisions for f_{ps} in unbonded strands (Eq. [4]) and f_{ps} in bonded strands (Eq. [10]) can be applicable to beams with combined strands. When using the Consolazio et al.⁵¹/Frosch et al.⁷⁷ method in Fig. 4, the 2024 AASHTO LRFD specifications⁸ requirement for $f_{ps} \leq f_{py}$ was not enforced.

Stress in bonded strands for beams with combined strands f_{psb} All four methods evaluated (the 2024 AASHTO LRFD specifications,⁸ Brenkus,⁴⁷ Unal,⁵³ and Consolazio et al.⁵¹/Frosch et al.⁷⁷) suggest using Eq. (10) of the 2024 AASHTO LRFD specifications to predict stress in bonded strands for beams with combined strands f_{psb} . This is a reasonable approach because the basis of Eq. (10) is strain compatibility, which remains valid for bonded strands in beams with combined strands. Only 11 tests (out of the 39 total tests) reported measured f_{psb} ; therefore, a comparison of predictions to measurements for bonded strands from the 11 tests is not provided here but can be found elsewhere.⁸⁷

The prediction of f_{psb} by the 2024 AASHTO LRFD specifications⁸ had a mean of 0.94, R of 0.64, and $RMSE$ of 200 MPa (29 ksi), showing that f_{psb} can be predicted reasonably well by the AASHTO LRFD specifications method. The %Ratio < 1.0 of the AASHTO LRFD specifications was 73%, indicating most measured values were less than code predictions, that is, 73% of cases were unconservative. The other methods had very similar predictions with small differences stemming from f_{psu} calculations affecting the neutral axis depth in Eq. (10).

Overall stress in combined strands The 2024 AASHTO LRFD specifications⁸ and Brenkus⁴⁷ methods recommend the use of an overall average stress f_{ps} , which is calculated as the weighted average of the bonded strand stress f_{psb} and the unbonded strand stress f_{psu} based on their areas. The bonded strand stress f_{psb} was calculated according to the 2024 AASHTO LRFD specifications for all specimens because only a few studies (11 out of 39 tests) reported the measured f_{psb} and, as

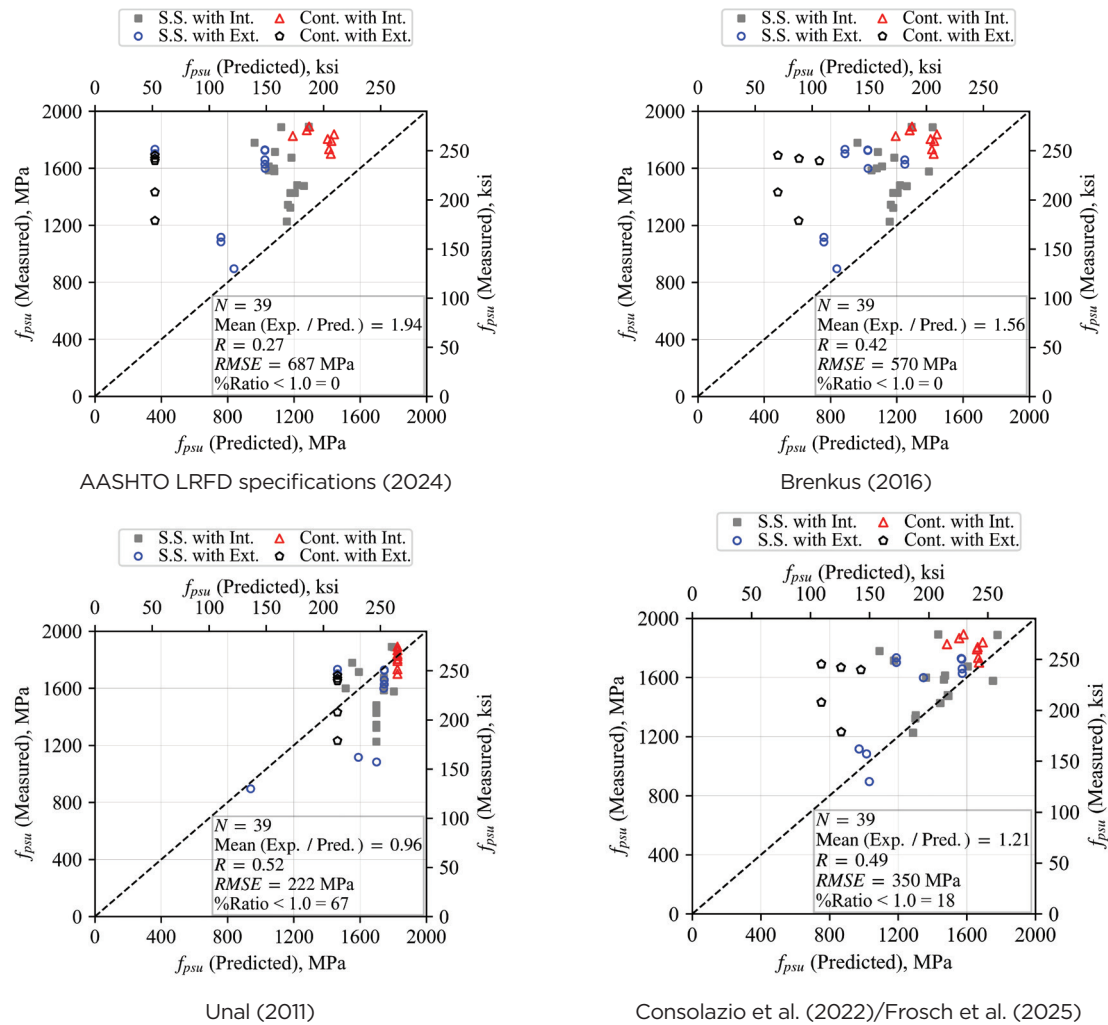


Figure 4. Predicted compared with measured unbonded strand stress f_{psu} for beams with combined strands. Note: AASHTO LRFD specifications (2024) = the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*, 10th edition; Cont. = continuous; Exp. = experimental; Ext. = external strands; Int. = internal strands; N = number of data points; Pred. = predicted; R = correlation coefficient; RMSE = root mean square error; S.S. = simply supported. 1 MPa = 0.145 ksi.

previously discussed, due to strain compatibility, the bonded strand stress can be calculated well with this method. **Figure 5** compares the average f_{ps} in combined strands as measured by the experiments and as predicted by the four methods. When calculating c for Eq. (10) as $c = (A_{psb}f_{psb} + A_{psu}f_{psu})/\alpha_1\beta_1f'_cb$ the 2024 AASHTO LRFD specifications⁸ and Brenkus⁴⁷ methods recommend the unbonded strand stress f_{psu} to be f_{pe} , while Consolazio et al.⁵¹/Frosch et al.⁷⁷ recommend the unbonded strand stress f_{psu} to be calculated using Eq. (4). Unal⁵³ does not address the calculation of c ; therefore, for this method, c was calculated based on force equilibrium.

Although Fig. 4 indicates that the Unal⁵³ method had the lowest RMSE for f_{psu} predictions, Fig. 5 shows the Consolazio et al.⁵¹/Frosch et al.⁷⁷ method had the lowest RMSE for the f_{ps} predictions. The difference is caused by the interdependency of c , f_{psb} , and f_{psu} . While both methods

used the 2024 AASHTO LRFD specifications⁸ equation for f_{psb} , different values of f_{psu} predicted by the two methods led to different values of c , which changed the f_{psb} and overall f_{ps} predictions. The method by Consolazio et al.⁵¹/Frosch et al.⁷⁷ had an f_{ps} prediction mean value close to 1.0 (1.06), the largest R (0.83), and the lowest RMSE (134 MPa [19.43 ksi]), indicating that the use of the AASHTO LRFD specifications equations for unbonded strands (Eq. [4]) and for bonded strands (Eq. [10]) is more accurate than any other prediction method for beams with combined strands.

Conclusion

The findings based on this research are as follows:

- For beams with only unbonded strands the following was observed:

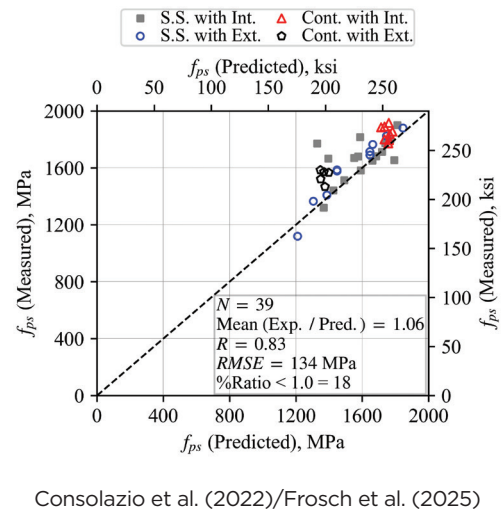
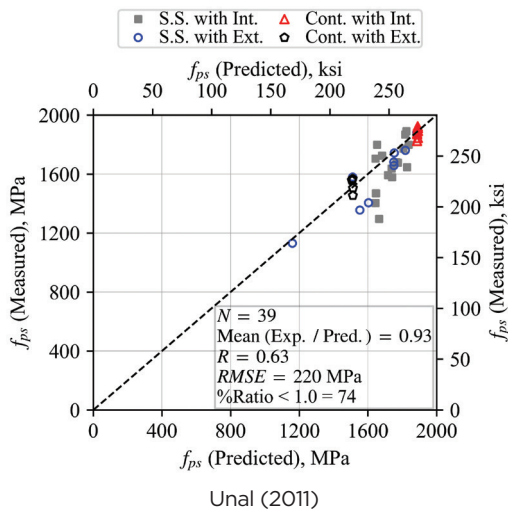
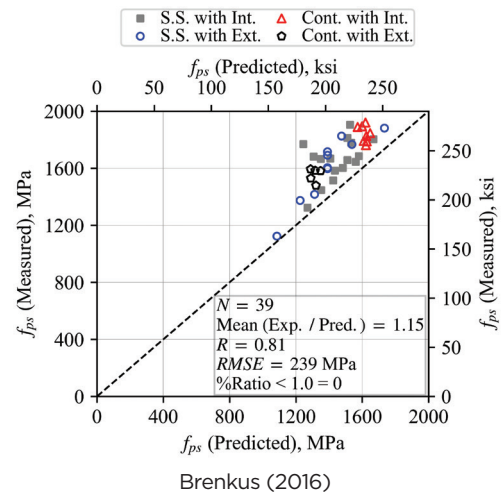
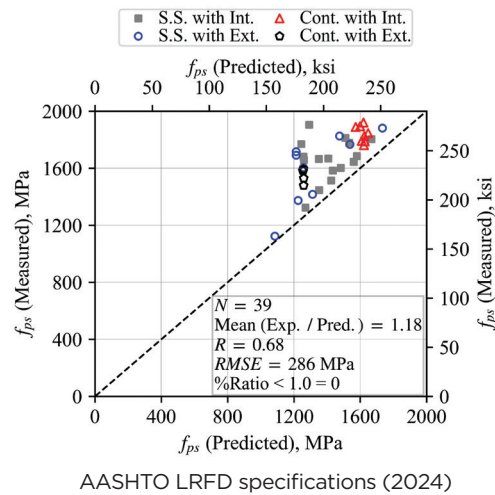


Figure 5. Predicted compared with measured average f_{ps} in combined strands.

Note: AASHTO LRFD specifications (2024) = the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*, 10th edition; Cont. = continuous; Exp. = experimental; Ext. = external strands; f_{ps} = stress in strands at the nominal flexural capacity of beams; Int. = internal strands; N = number of data points; Pred. = predicted; R = correlation coefficient; $RMSE$ = root mean square error; S.S. = simply supported. 1 MPa = 0.145 ksi.

- The *Canadian Highway Bridge Design Code* (CSA S6:1973) had the highest mean ratio of experimentally measured to predicted f_{ps} (1.42) along with the highest $RMSE$, indicating that assuming f_{ps} to be equal to the effective prestress f_{pe} (the lower bound) will result in conservative but inaccurate predictions. Despite being derived based on different approaches, the 2024 AASHTO LRFD specifications,⁸ 1994 AASHTO LRFD specifications,⁶⁵ and ACI 318-25⁷ had similar statistical measures with the mean of the ratio of experimental to predicted values ranging between 1.08 to 1.10; all three predicted unbonded strand stress well.
- All prediction methods provided poorer predictions of f_{ps} for continuous beams evidenced by their generally lower coefficients of correlation R and the higher $RMSE$ values compared to simply supported beams.

This may be due to the relatively limited test data (30 tests) on continuous beams compared with simply supported beams (266 tests).

- For beams with combined (unbonded and bonded) strands:
 - Predictions of unbonded strand stress in beams with combined strands had a larger scatter and higher conservatism when compared to predictions of strand stress for beams with unbonded strands only. This could be explained by the smaller number of tests and studies conducted on beams with combined strands and the fact that some prediction methods predict combined strand stresses using equations originally developed for unbonded strands. Methods by Unal,⁵³ Consolazio et al.,⁵¹/Frosch et al.⁷⁷ provided better predictions compared with those of the 2024

AASHTO LRFD specifications⁸ and Brenkus,⁴⁷ although the Unal⁵³ method was the least conservative with the majority of the predictions greater than the measurements. The lower level of accuracy provided by the 2024 AASHTO LRFD specifications⁸ and Brenkus⁴⁷ may be due to neglecting unbonded strand stress in all cases for the 2024 AASHTO LRFD specifications⁸ and in some cases for Brenkus.⁴⁷ The smaller dataset used to develop the Brenkus⁴⁷ equations may also have increased the prediction error.

- Consolazio et al.⁵¹/Frosch et al.⁷⁷ provided the most accurate predictions for the overall (average) stress in combined strands, indicating that the equations provided in the AASHTO LRFD specifications (2024)⁸ for beams with only bonded strands (Eq. [10]) and for beams with only unbonded strands (Eq. [4]) are applicable to beams with combined strands. Unlike the 2024 AASHTO LRFD specifications⁸ predictions, which were all conservative, overall stress predictions of the Consolazio et al.⁵¹/Frosch et al.⁷⁷ method were slightly unconservative for some beams.
- Available test data were limited for beams with combined strands (39 specimens) compared with beams with unbonded strands only (296 specimens). Additional tests are needed to further improve the development and evaluation of methods to predict strand stresses for beams with combined strands (bonded and unbonded).

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Notation

A_{ps} = prestressing strand area

A_{psb} = area of bonded prestressing strands

A_{psu} = area of unbonded prestressing strands

A_s = area of mild steel reinforcement

b = beam width

b_w = beam web width

c = neutral axis depth

c_e = neutral axis depth at ultimate, assuming f_{ps} in unbonded strands is equal to f_{pe}

d_{ps} = distance from the extreme compression fiber to the centroid of all strands

d_{psb} = distance from the extreme compression fiber to the centroid of bonded strands

d_{psu} = distance from the extreme compression fiber to the centroid of unbonded strands

d_{ps0} = initial distance from the extreme compression fiber to the centroid of external strands

e_m = strand eccentricity at midspan

e_{ps} = eccentricity of unbonded strands

E_{ps} = modulus of elasticity of prestressing steel

f = a ratio of span length to distance between loading points = ∞ for a single concentrated load = 3 for two-point loads applied at third points across a span = 6 for uniform loading

f'_c = compressive strength of concrete

f_{pe} = effective stress in prestressing strands

f_{ps} = stress in strands at the nominal flexural capacity of beams

f_{psb} = stress in bonded strands at the nominal flexural capacity of beams with combined strands

f_{psu} = stress in unbonded strands at the nominal flexural capacity of beams with combined strands

f_{pu} = ultimate strength of prestressing steel

f_{py} = yield strength of prestressing steel

f_y = yield strength of mild steel reinforcement

h = beam depth

k = factor to account for prestressing steel material

k_s = constant to account for second-order effects by external strands

L = span length

L_d = distance from beam support to deviation point

L_e	= effective strand length	ψ	= scaled plastic hinge length
L_i	= strand length between anchorages	Ω_u	= bond reduction coefficient
L_p	= plastic hinge length		
L_1	= length of loaded span affected by strand		
L_2	= length of strand between end anchorages		
n_p^-	= the number of negative moment plastic hinges		
n_p^+	= the number of positive moment plastic hinges		
N	= number of data points		
N_p	= load parameter		
N_s	= number of plastic hinges at support in an assumed failure mechanism crossed by the strand between anchorages		
R	= correlation coefficient		
$RMSE$	= root mean square error		
R_s	= stress increase reduction factor to account for second-order effects by external strands		
S_d	= distance between two symmetrically placed deviators		
Z	= shear span		
α_1	= uniform compressive stress magnitude factor in the equivalent stress block		
β_1	= concrete compressive stress block reduction factor		
γ	= reduction factor to account for bonded and unbonded prestressing strand areas		
δ	= change in strand length at a hinge location		
Δ	= span deflection		
$\Delta\epsilon_{ps}$	= strain increase in prestressing strands		
ϵ_{cu}	= ultimate compressive strain of concrete		
η	= parameter based on loading type and strand profile		
θ	= angle of rotation at the hinge		
ϕ	= curvature in Eq. (3)		
ϕ	= parameter based on strand type in He and Liu (2010) study		

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Abstract

This paper evaluates procedures from design specifications and the literature for predicting strand stress at the nominal flexural capacity of prestressed concrete beams with combined (bonded and unbonded) strands and with only unbonded strands. Flexure test data were compiled for 39 beams with combined strands and 296 beams with only unbonded strands. For beams with only unbonded strands, design specifications procedures based on a bond reduction coefficient, collapse mechanism, or empirical approaches had similar accuracies and predicted the unbonded strand stress well. While conservative, assuming the unbonded strand stress is equal to the effective prestress resulted in inaccurate predictions. For beams with combined strands, predictions were the most accurate when unbonded strand stresses were predicted using the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications* 10th edition expression meant for beams with only unbonded strands and when bonded strand stresses were predicted using the *AASHTO LRFD specifications* 10th edition expression meant for members with only bonded strands.

Keywords

Bending, bonded, combined, external strands, hybrid, strand stress increase, unbonded.

Review policy

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